نموذج إجابة الرياضيات الهندسية الفرقة
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Model Answer Eng. Math. (2b)

Date 10/6/2015 Code: PME1205 First year civil

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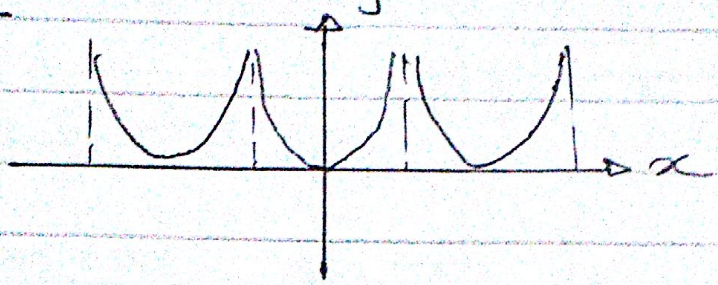
Question No. (1)

(a)

$F(x) = x^2$

$-1 \leqslant x \leqslant 1$



Even fn. $\longrightarrow$ cosine series, $b_n = 0$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = 2\int_0^1 x^2\,dx = \frac{2}{3}x^3\Big|_0^1$$

$$a_0 = \frac{2}{3}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = 2\int_0^1 x^2\cos n\pi x\,dx$$

$$a_n = 2\left[\frac{x^2\sin(n\pi x)}{n\pi} + \frac{2x\cos(n\pi x)}{(n\pi)^2} - \frac{2\sin(n\pi x)}{(n\pi)^3}\right]_0^1$$

$$a_n = \frac{4(-1)^n}{(n\pi)^2}$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos n\pi x$$

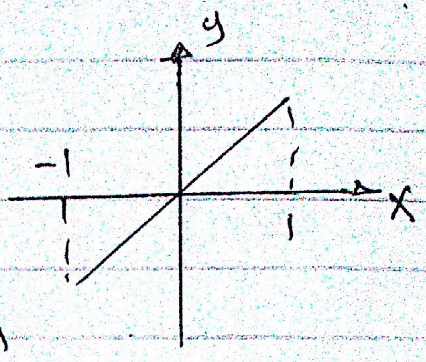
$$F(x) = \frac{1}{3} + \sum_{1}^{\infty}\frac{4(-1)^n}{(n\pi)^2}\cos(n\pi x)$$

(b) $f(x) = x \quad , \quad 0 < x < 1$

- Sine Series

$$a_0 = a_n = 0$$

$$b_n = \frac{2}{L}\int_0^L f(x) \sin\frac{n\pi x}{L}\, dx$$



$$= 2\int_0^1 x \sin n\pi x \, dx$$

$$= 2\left[\frac{-x\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2}\right]_0^1$$

$$b_n = \frac{2}{n\pi}(-1)^{n+1}$$

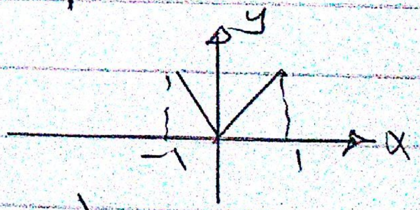
$$f(x) = \sum_1^\infty \frac{2}{n\pi}(-1)^{n+1} \sin n\pi x$$

- Cosine Series

$$b_n = 0$$

$$a_0 = 2\int_0^1 x\, dx = x^2\Big|_0^1 = 1$$

$$a_n = 2\int_0^1 x \cos n\pi x \, dx$$



$$= 2\left[\frac{x \sin n\pi x}{n\pi} + \frac{\cos n\pi x}{(n\pi)^2}\right]_0^1$$

$$= 2\left[\frac{(-1)^n}{(n\pi)^2} - \frac{1}{(n\pi)^2}\right]$$

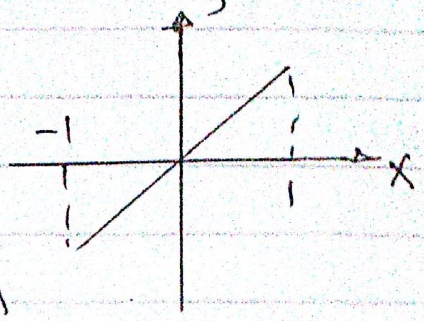
$$a_n = \frac{2}{(n\pi)^2}\left[(-1)^n - 1\right]$$

$$f(x) = \frac{1}{2} + \sum_1^\infty \frac{2}{(n\pi)^2}\left[(-1)^n - 1\right]\cos n\pi x$$

(b) $f(x) = x \quad , \quad 0<x<1$

— Sine Series

$$a_0 = a_n = 0$$



$$b_n = \frac{2}{L}\int_0^L f(x) \sin\frac{n\pi x}{L}\, dx$$

$$= 2\int_0^1 x \sin n\pi x \, dx$$

$$= 2\left[\frac{-x\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2} \right]_0^1$$

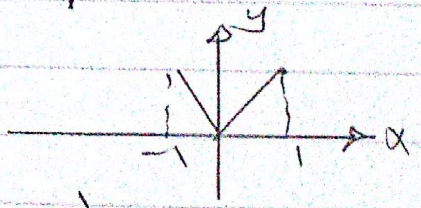
$$b_n = \frac{2}{n\pi}(-1)^{n+1}$$

$$f(x) = \sum_1^{\infty} \frac{2}{n\pi}(-1)^{n+1} \sin n\pi x$$

— Cosine Series

$$b_n = 0$$

$$a_0 = 2\int_0^1 x\, dx = x^2\Big|_0^1 = 1$$



$$a_n = 2\int_0^1 x \cos n\pi x \, dx$$

$$= 2\left[x\frac{\sin n\pi x}{n\pi} + \frac{\cos n\pi x}{(n\pi)^2} \right]_0^1$$

$$= 2\left[\frac{(-1)^n}{(n\pi)^2} - \frac{1}{(n\pi)^2} \right]$$

$$a_n = \frac{2}{(n\pi)^2}\left[(-1)^n - 1\right]$$

$$f(x) = \frac{1}{2} + \sum_1^{\infty} \frac{2}{(n\pi)^2}\left[(-1)^n - 1\right]\cos n\pi x$$

Question no. (2)

(a) - $F(t) = t \sinh 4t$

$$F(s) = (-1)^1 \frac{d}{ds}\left(\frac{4}{s^2-16}\right)$$

$$= -1 * \left[\frac{0 - 2s * 4}{(s^2-16)^2}\right]$$

$$F(s) = \frac{8s}{(s^2-16)^2}$$

- $F(t) = \sin^2 t$

$$= \frac{1}{2}[1 - \cos 2t]$$

$$F(s) = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$$

$$F(s) = \frac{2}{s(s^2+4)}$$

(b) - $L^{-1}\left[\frac{s+10}{s^2-16}\right]$

$$= L^{-1}\left[\frac{s}{s^2-16}\right] + \frac{10}{4} L^{-1}\left[\frac{4}{s^2-16}\right]$$

$$F(t) = \cosh 4t + \frac{5}{2}\sinh 4t$$

- $L^{-1}\left[\frac{e^{-20s}}{s^2+4}\right]$

$$= L^{-1}\left[\frac{1}{2}\,\frac{2}{s^2+4} * e^{-20s}\right]$$

$$F(t) = \frac{1}{2}\sin 2(t-20)\; u(t-20)$$

(c) $L[y'''] + 2L[y''] - L[y'] - 2L[y] = 0$

$s^3 y(s) - 2s - 2 + 2s^2 y(s) - 4 - s y(s) - 2y(s) =$

$$y(s) = \frac{2s + 6}{s^3 + 2s^2 - s - 2}$$

$$y(s) = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$Y(s) = \frac{5/3}{s-1} - \frac{1}{s+1} + \frac{1/3}{s+2}$$

$$y(t) = L^{-1}[Y(s)] = \frac{5}{3}e^{t} - e^{-t} + \frac{1}{3}e^{-2t}$$